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Equilibrium of Bolted Joints Subjected to In-Plane External Loading

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The objective of this article is to explain a new technique for determining the maximum load-bearing capability of a "bolted joint" subjected to "in-plane" external loading. The uniqueness of this technique lies in overcoming the present absence of a suitable stress/strain framework while still working conservatively within proper MIL-HDBK restrictions. This is accomplished by the (not unreasonable) hypothesis that "load" is actually distributed in proportion to available "bearing area," as opposed to "shear area," since this would appear to be a more valid assumption when aluminum members are joined by steel or titanium fasteners that are designed to be "bearing critical." In this article, aluminum structures are principally used for example only and do not serve as a restriction. The consequence of the bearing stress/strain ratio that is derived is that a given bolt group can be shown to have increased ultimate load and moment capability over the classical solution, which assumes a maximum capability when the bearing stress on the worst loaded fastener has reached f_{br} ultimate.

Introduction

THE major portion of this article lies in the construction of 13 specific figures in the form of viewgraphs. The motivation for doing this is to provide both the teacher and students, in a strength of materials and structures design course, an effective "teaching aid" about the nontrite problems that must be considered in designing safe ultimate loads on light structures such as, e.g., aircraft assembly components, when decisions must be made as to how they will be fastened together.

It is hoped that this "a picture is worth a thousand words" approach will both stimulate an in-depth discussion and critique about the principles involved, thereby developing a more thorough understanding of them, and provide an alternative approach that may be found to be of enough interest and controversy to warrant further verification by experimentation.

It is also hoped that by further examination of this approach, an improved standard of design for bolted joints will evolve.

In the interests of maintaining good structural integrity, responsible design engineers, in the pursuit of developing good "margins of safety" that guarantee this integrity, will often err on the side of caution to ensure that it is maintained. Consequently, for the multitude of different component parts that are required to be bolted together for aircraft subassemblies, e.g., many engineers are taught that the traditional "elastic" approach is the best to use when designing the placement, or the geometry of the "array," of such fasteners (bolts). This traditional approach says that applied loads are always distributed in proportion to "fastener shear area" and the distance from the "c.g." of the fastener array.[1]

The real quarrel with this approach is that it can often be too conservative when a re-evaluation by a "plasticity analysis" can show that the margins of safety so derived are larger than necessary. This is especially unfortunate when not only costly redesign configurations are then called for, but when the final design is adopted it is less efficient or functional than the design that was scrapped.

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Initiation of the Procedure

In the viewgraphs that follow, Fig. 1 raises two specific questions concerning the validity of the traditional approach. How valid is it, and why change it if it works so well? Here, it is important for the reader to consider the fact that aluminum alloy members are often joined by both steel and titanium fasteners that are specifically designed to be bearing critical. Consequently, Fig. 1 is displayed first for the purpose of focusing our attention as insightfully as possible on the problem and then for considering whether bearing area is a more valid assumption to make, as opposed to shear area, in determining the actual proportionality that load should be distributed.

Understanding the relevance of Fig. 1, Fig. 2 is designed to not only make the overall objective of this article clear, the determination of what might be considered the real maximum load-bearing capability of a bolted joint, but to make the reader understand the critical problem standing in the way of achieving the objective, the absence of a suitable MIL-HDBK reference (for aluminum alloy products) upon which to build a bearing stress/strain analysis, and the proposed solution for overcoming this difficulty (making use of the fact

The Classical Solution for Determining Load Distribution on Bolted Joints

For all bolted joints, many of us have been taught, in the traditional elastic approach, that 'applied' loads are always distributed in proportion to 'fastener shear area' and the distance from the 'centre of gravity' of the fastener array.

Question 1

Is this a valid proposition?

Answer

Yes - We build excellent aeroplanes, bridges, and skyscrapers by using it.

Question 2

If this proposition works so well, why change it?

Answer

The hypothesis that load is actually distributed in proportion to available 'bearing area' (as opposed to 'shear area') would appear to be a more valid assumption when aluminum alloy members are joined by steel or titanium fasteners which are designed to be 'bearing critical'.

Fig. 1 Critical questions and answers.

Objective

To develop a technique for determining the maximum load bearing capability of a "Bolted Joint" subjected to "in-plane" external loads.

Problem in Achieving Objective

Absence of a suitable framework of Stress / Strain diagrams for "bearings" in MIL - HDBK 5E (for Aluminum Alloy Products) to support development of technique.

Proposed Solution of Problem

A) Given that for a typical aluminum alloy such as 7075-T6,

-strain at $F_{ty} \simeq 1\%$
-strain at $F_{tu} \simeq 10\%$ } which implies a 10:1 ratio for tension

(Ref: MIL - HDBK 5E, Figs. 3 7 4 1 6 (n),(o),(p))

We will assume that this same ratio (10:1) is a good approximation for "bearing".

B) Using (A) as a fundamental premise, to then construct a mathematical system for determining maximum load bearing capability of a "Bolted Joint". This process is referred to as "Ripple" (to be described).

Fig. 2 Objective and proposed solution.

Approach (For each fastener)

Since 0.67 $Fbr_u < Fbr_y$ for most engineering materials in use then if, for the most heavily loaded fastener, we establish a "cut-off" at design limit load of:

0.67 x Ultimate Load

So that:

Fbr Applied $\not> Fbr_y$

Consequences

This means a bolt group (which is designed to be bearing critical) can be shown to have:

1. Increased Applied Load Capability
2. Increased Moment Capability

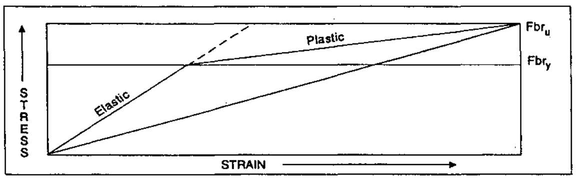


Fig. 3 Approach and consequences.

that the general ratio of (F_t ultimate:F_t yield) implies a (10:1) ratio for "tension" for typical aluminum alloy products, and thus, would appear to be an appropriate ratio for "bearing" ultimate and bearing yield also.

Approach and Consequences

Since it is always the case that $0.67(Fbr_u)$ is always strictly less than Fbr_y for most engineering materials in use, then the "integrity" of our bearing area approach seems to be well-maintained when we deliberately establish a "cut-off" at design limit load of $0.67 \times$ ultimate load, so that Fbr applied will always be strictly less than Fbr_y, which makes the concept of "plastic" behavior legitimate.

Figure 3 amplifies this approach, giving us the resulting consequence that bolt groups that have been designed to be bearing critical will have both 1) increased applied load capability and 2) increased moment capability.

This "increase" is pictorially represented by the triangle "area" characterized by the intersections of the lines labeled elastic, plastic, and the diagonal line terminating at Fbr_u. In fact, the diagram of Fig. 3 is especially important since it is only just the area below the "diagonal line" that is ever considered in the traditional elastic approach of bolt group analysis (the "upper triangle area," which depicts the earlier increase, is ignored).

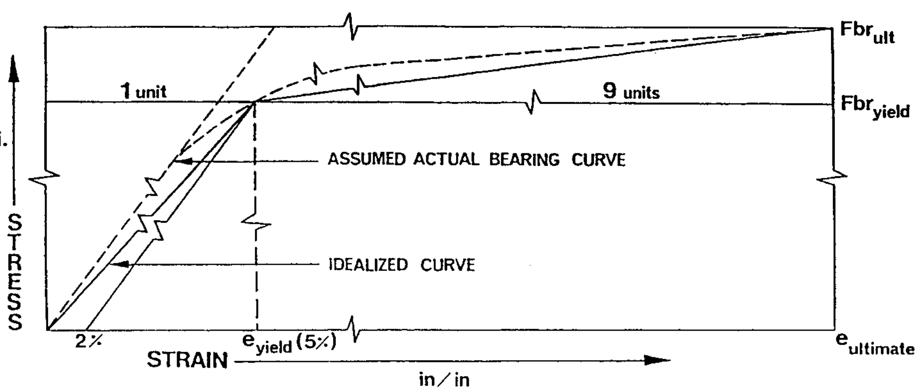
Hypothesized Bearing Stress/Bearing Strain Relationship

Figure 4 portrays the essence of the "new" approach by suggesting that the stress/strain ratio might best be expressed as

$$e_{\text{yield}}/e_{\text{ultimate}} = 1/10$$

where, by virtue of the four definitions of: modulus of elasticity E, strain yield (Fbr_y/E), strain ultimate ($10 \times$ strain yield), and the measure of "strain" itself (% elongation/nominal hole diameter), with hole elongation set at 2% gauge length, we can now construct a legitimate framework of stress/strain diagrams analogous, e.g., to those found in MIL-HDBK 5E [Figs. 3.7.4.1.6 (n), (o), and (p) (Ref. 2)].

In fact, the pictorial representation of Fig. 4 appears to resolve the problem of achieving our objective, as stated in



DEF 1 E = Slope of Straight Line Portion of Bearing Stress / Bearing Strain Relationship (Modulus of Elasticity of Material)

DEF 2 $e_{yield} = \frac{Fbr_y}{E}$

DEF 3 $e_{ultimate} = 10 \times e_{yield}$

DEF 4 $\text{STRAIN} = \frac{\text{Percentage of Elongation}}{\text{Nominal Hole Diameter}}$

STRESS-STRAIN RATIO
Suggestion

$$\frac{e_{yield}}{e_{ultimate}} = \frac{1}{10}$$

Fig. 4 Stress/strain relationship.

We initialize this process by the following simplifying assumptions:

① The fasteners are infinitely rigid. That is, fastener bearing, shear and bending deflections can be ignored.

② Axial, shear and bending deflections in the supporting and reacting plates at the joint are small and can be ignored.

③ When joint displacement and/or rotation occurs, all radial lines remain straight.

④ When external loads are applied, all deflections at the joint are due to local bearing deflections at the fasteners.

⑤ $e_{yield} \simeq$ 5% of nominal hole diameter "d" (i.e. $e_{yield} = 0.05d$) giving "ultimate strain at failure" 50% of d (i.e. $e_{ult} = 10e_{yield} = 0.5d$)

⑥ Edge distance conditions are resolved by imposing a "cut-off" value of "fastener load allowable" calculated from the "shear-out" capability the material between the hole and the edge of the material.

Fig. 5 Necessary assumptions.

Fig. 2 (an absence of a suitable framework of stress/strain diagrams for bearing in MIL-HDBK 5E) by supplying such a diagram for hypothetical consideration.

In support of the previous hypothesis, Fig. 5 lays out a set of six simplifying assumptions that are designed to eliminate excess computational complexities, which, if admitted at this stage, would likely unnecessarily cloud the essence of our theory, which itself is more important to convey as clearly and as succinctly as possible. These assumptions are as follows:

1) The fasteners are infinitely rigid and that, therefore, fastener bearing and shear and bending deflections can be ignored.

2) Axial shear and bending deflections in the supporting and reacting plates at the joint are small and can be ignored.

3) When joint displacement and/or rotation occurs, all radial lines remain straight.

4) When external loads are applied, all deflections at the joint are due to local bearing deflection at the fastener.

5) e_{yield} is approximately equal to 5% of nominal hole diameter "d" (i.e., $e_{yield} = 0.05d$) giving "ultimate strain at failure" 50% of d (i.e., $e_{ult} = 10e_{yield} = 0.5d$).

6) Edge distance conditions are resolved by imposing a cutoff value of "fastener load allowable" calculated from the "shear-out" capability of the material between the hole and the edge of the material.

Theoretical Concept of Ripple for Measuring Bolt Group Equilibrium or Failure

In keeping with the desire to convey as simplistic an understanding of the theory as possible, Figs. 6–10 consider the "transition stages" that two simple flat plates bolted together would theoretically "pass through," in a real time mode, for some moving (flight) condition, when a load of "p" pounds is applied in-plane externally to an initial fastener of an array of fasteners (bolts).

For real clarity, we make very simplifying assumptions again. Here we consider a flat plate, as described in Fig. 6, to be of uniform density, of constant thickness, of having a bearing ultimate stress Fbr_u, and a bearing yield stress Fbr_y of p pounds, of being fastened together by a horizontal row of four fasteners (all of common hole diameter, pitch, and having no "short-edge distance" problem), and bolted to another plate of similar dimensions that is held rigid, but whose bearing yield stress is considerably greater than that of the "first" plate. We also assume that the fasteners have unit-bearing area, and thus "stress" is equated to "pounds."

Figure 7 simply directs our attention to the beginning of the "ripple process," the initial stage, where the four fasteners are displayed with no internal bolt group reaction having yet

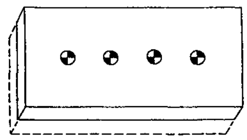
The Concept of a "Ripple" (an Example)

Consider a flat plate:

1. of uniform density
2. of constant thickness
3. having a bearing ultimate stress (Fbr_u) and a bearing lead stress (Fbr_y) of p lbs
4. that by means of a horizontal row of four (4) fasteners which:
   a) are of common hole diameter
   b) are of common pitch
   c) have no 'short edge distance' problem

is bolted to another plate:

i) of similar physical dimensions
ii) that is held rigid
iii) whose bearing yield stress (Fbr_u) is 'considerably' greater than the bearing stress (Fbr_y) of the first plate



Note: For convenience we will assume that fasteners have unit bearing area and thus 'stress' will be equated to lbs.

Fig. 6 Conceptual considerations.

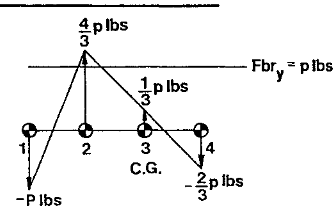
Initial State

Four Fasteners - 0 Load

Fig. 7 Initial state.

A load of P lbs is applied to the first (I) fastener which causes the:

First Ripple State (As Illustrated)



With: Moment balance occuring because

$$(1\times\frac{4}{3}p)+(2\times\frac{1}{3}p)-(3\times\frac{2}{3}p)=(\frac{4}{3}+\frac{2}{3}-\frac{6}{3})p=0$$

Load balance occuring because

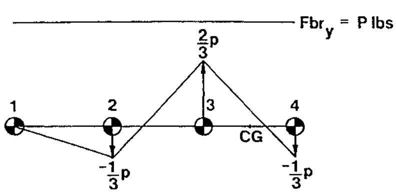
$$-(p)+(\frac{4}{3}p)+(\frac{1}{3}p)-(\frac{2}{3}p)=(-1+\frac{4}{3}+\frac{1}{3}-\frac{2}{3})p=0$$

But: Fastener two (2) must now 'shed' $\frac{1}{3}$p lbs onto fasteners three (3) and four (4) to avoid plasticity.

Fig. 8 First ripple state.

A load of $\frac{1}{3}$p lbs is 'shed' from fastener two (2) to the remaining fasteners three (3) and four (4) which causes the

Second Ripple State (As Illustrated)



With: Moment balance occuring because

$$(1\times\frac{2}{3}p)+(2\times\frac{1}{3}p)=(\frac{2}{3}-\frac{2}{3})p=0$$

Load balance occuring because

$$-(\frac{1}{3}p)+(\frac{2}{3}p)-(\frac{1}{3}p)=(-\frac{1}{3}+\frac{2}{3}-\frac{1}{3})p=0$$

Fig. 9 Second ripple state.

If we now combine the Initial State, First Ripple State, and Second Ripple State, by linear addition a Resultant Equilibrium State is achieved:

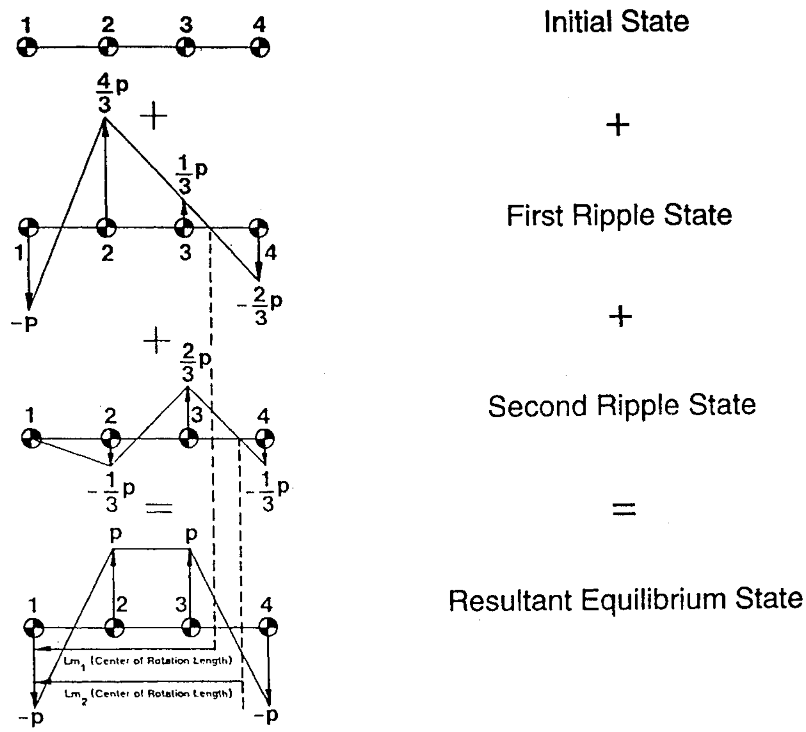


Fig. 10 Overview of combined ripple states.

taken place because of the absence of any in-plane external loading.

However, as illustrated in Fig. 8, when a load of p pounds is applied "vertically" and in-plane to the first fastener of the four-fastener array (which is horizontal to this "applied" load), a natural condition of both moment and load balancing begins to occur on all fasteners, starting with the first fastener, since it received the load p, which was initially applied to it, in order to restore (if possible) equilibrium.

Here, we note momentary failure in the "second" fastener since its load-bearing capability has exceeded its bearing yield capability by one-third. If fastener number two was to stand alone (i.e., fasteners three and four did not exist), it must surely rupture or else cause the first plate to begin to rotate counterclockwise about the second plate with the "center of rotation" being shifted to the right of the c.g.'s position, which lies midway between the first two fasteners, to create an equal and opposite "shedding" of load.

However, what will actually occur is that fastener number two attempts to shed its excess load first "onto" fasteners three and four, the ripple process, before the bolted structure ruptures.

This reaction triggers the second ripple process, as depicted in Fig. 9, with a similar moment and load rebalancing, as illustrated, to restore equilibrium. Here, fastener three is seen to absorb a load excess of one-third p pounds (which increases its load-bearing capacity from one-third p pounds to two-thirds p pounds), while diminishing the load-bearing capacity of the fourth fastener by the same amount. This again is the result of an equal and opposite reaction caused by a "shifted" center of rotation.

It is this second ripple that restores a state of equilibrium for, as depicted in Fig. 10, if we now combine the initial state (Fig. 7), the first ripple state (Fig. 8), and the second ripple state (Fig. 9) by linear addition of all "load propagations" separately on each fastener, a load balance of

$$-P + [(4/3)P - (1/3)P] + [(1/3)P + (2/3)P] + [(-2/3)P + (-1/3)P] = 0$$

or, more simply

$$-P + P + P - P = 0$$

has occurred. This is significant for two reasons. None of the fasteners has gone plastic, and a plastic condition did momentarily occur but was "rippled" out.

Geometry of a Bearing Stress/Bearing Strain Relationship

In order to understand the computational aspects that are required to evaluate the load-bearing capabilities of fasteners under the proposed plasticity analysis, we refer to the geometry of Fig. 11, which portrays the "four fastener" example described earlier, and where we define the following:

u_{01} = a plasticity reference point associated with fastener one
p_{12} = the pitch between fasteners one and two
u_{02} = the measure of stress the second fastener had to sustain in the plastic zone when it exceeded yield stress by one-third p pounds during the "first ripple"
p_{13} = the pitch between fasteners one and three
u_{13} = the measure of stress the third fastener had to sustain when an increase of one-third p pounds was added to it during the "second ripple," but which still kept the total load that the third fastener had to sustain within the elastic zone
x = one-tenth "gauge length" analogizing a bearing stress (10:1) ratio with that found in tension (F_t ultimate: F_t yield) for typical aluminum products
$9x$ = gauge length minus x
$(u - y)/9$ = similar triangle proportionality to $u - y$
u = ultimate stress
y = yield stress
$k_u y$ = line intercept with straight line plastic equation
Lm = center of rotation

Carefully examining Fig. 11, it may be seen that the "slope" of the "plastic equation" is $(u_{01} - k_u y)/Lm$, while the slope of the "elastic equation" is this constant again multiplied by the "transition factor" $9y/(u - y)$. This is because the proportionality factor $(u - y)/9$ when multiplied by $9y/(u - y)$ equals the yield y itself.

Figure 12 portrays the plastic and the elastic slope measures $m(P)$ and $m(E)$ respectively, with the sliding transition factor K_x by which fasteners move from the "plastic line" to the "elastic line" or back again, depending upon the ripple effect causing fastener bearing loads to shift from one state of stress to another.

Figure 12 also clearly shows that an elastic state of stress may be determined as a point on the elastic equation $y = mx$ [where $m = m(E) = (u_{01} - k_u y)/Lm$ and $x = Lm - p_{13}$], while a plastic state of stress may be determined as a point on the plastic (line intercept) equation $y = mx + b$ [where we define $m = m(P) = m(E)K_x$, with the transition factor $K_x = 9y/(u - y)$ and x and b defined as $Lm - p_{12}$ and $k_u y$, respectively].

The "solution of u_{02}," as illustrated, shows the plastic state point value of the second fastener after the first ripple, while the "solution of u_{13}" shows the elastic state point value of the third fastener after the second ripple.

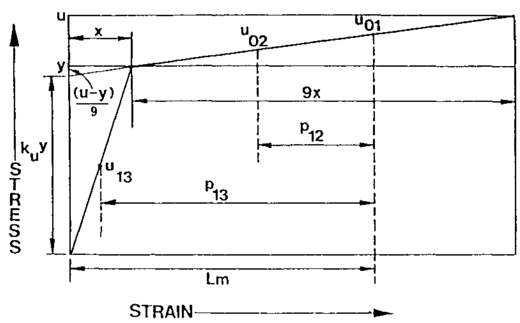


Fig. 11 Geometry of four fastener example.

Figure 13 is actually a stand-alone figure, separate from the previous 12 figures. Its intent is to generate personal perspectives about the behavior of materials themselves as opposed to the fasteners that hold them together.

The questions to be asked are those concerning "strain hardening" and "permanent set" when external loads are either applied or relaxed. Here, a hypothetical example is given to illustrate such a condition on a group of elements that could occur when an external load is applied, then relaxed.

Of particular relevance for serious discussion is the extension of our two-dimensional "line equation" procedure to a three-dimensional structure, as it appears relatively easy when placed in the context of three-dimensional vectors and employing vector geometry to get the correct "resultants."

Consequently, our line approach would appear to be a very simplistic and, possibly, a quite accurate way to evaluate internal loadings of point elements in a three-dimensional structure.

Discussion

While our own investigations with the materials in common use today appear to bear out the reasonableness of this new approach, it is essential to recognize the necessity that continued research into the feasibility of adopting this approach to bolt group analysis should take place. In so doing, a thorough investigation of the hypothesis should take place. That is, the hypothesis that the stress/strain ratio is best expressed as

$$e_{\text{yield}}/e_{\text{ultimate}} = 1/10$$

(as a result of the assumption that $e_{\text{yield}} = Fbr_y/E$ and $10 \times e_{\text{yield}} = e_{\text{ultimate}}$) and which is made by virtue of the fact that for most materials in common use today, it is also approximately true that

$$Ft_{\text{yield}}/Ft_{\text{ultimate}} = 1/10$$

In particular, the assumption of bearing yield strain being set at $0.05d$ with ultimate bearing strain set at $0.5d$ should be

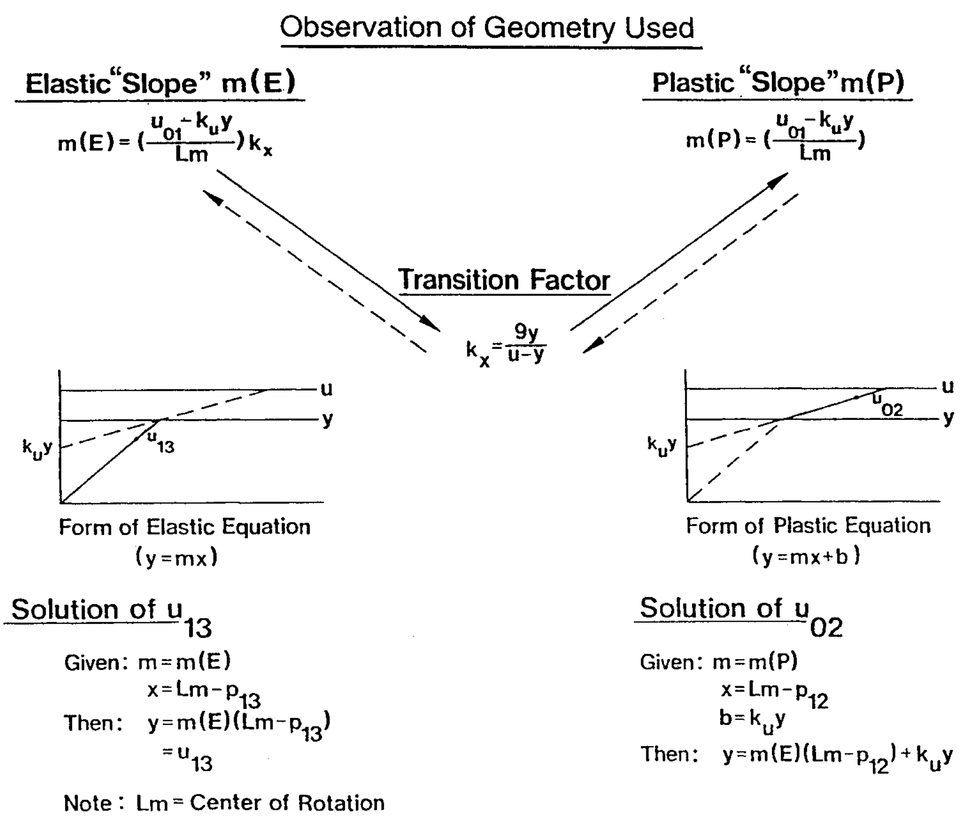


Fig. 12 Elastic/plastic slope relationship.

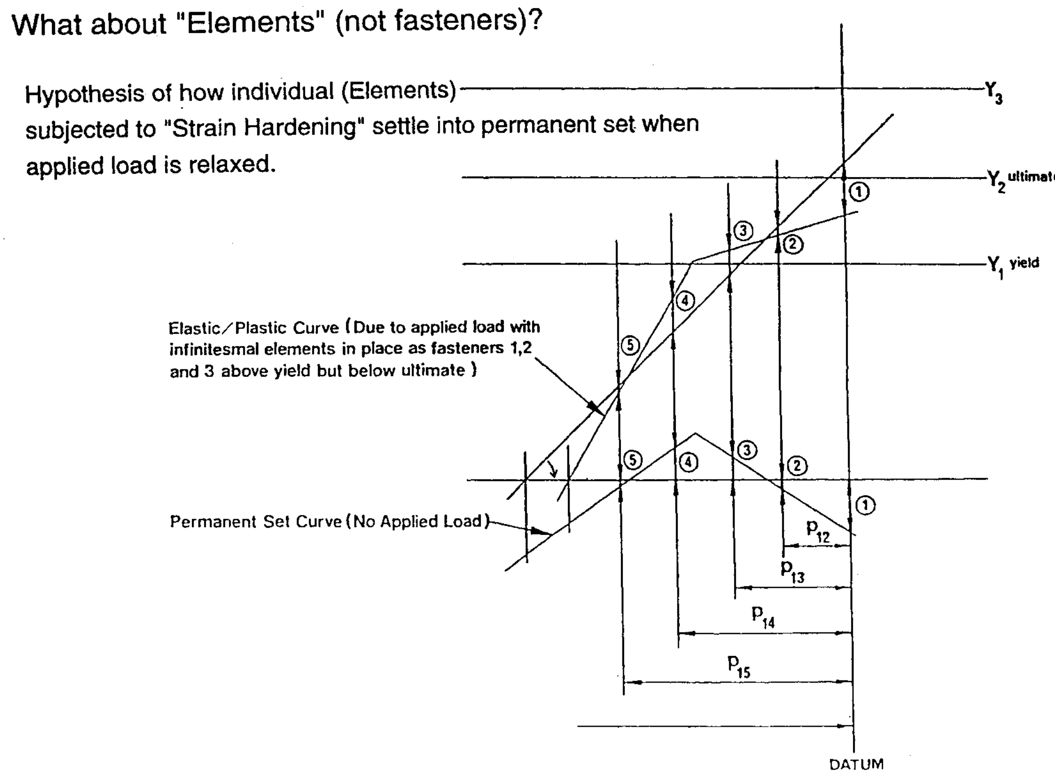


Fig. 13 Strain hardening and permanent set.

verified as exhaustively as possible. This seems totally reasonable, e.g., if we are also to include new materials in our designs as they enter the market.

We should also ask ourselves some seemingly not too unreasonable questions such as the following.

1) Is a tensile yield strain of 0.01 possibly a little high? Leading to the next question:

2) Is a measure between 0.001–0.002 perhaps more reasonable for a tensile yield strain?

3) Would a measure between 0.1–0.2 in gauge length not possibly be a better expectation for ultimate strain in tension?

4) Could a 10:1 (ultimate:yield) ratio for bearing be inaccurate because, in fact, bearing is a combination of tension, compression, and shear behavior about the circumference of a fastener's hole and that, therefore, basing our 10:1 ratio on tension behavior, as we have done, may be erroneous?

5) Because the yield offset criteria between net tension and bearing differ, to what extent then, if any, would this affect our hypothesized 10:1 ratio?

We ask these questions for the purpose of developing a few good starting positions upon which to base some exacting investigations. This is important because irrespective of the research and development programs that we have engaged in, where such types of questions have been examined in some depth, our findings have to be seriously verified by others.

This is necessary, for the question we are now asking other researchers to confirm is the following:

6) Since $0.67 \times Fbr_{\text{ultimate}}$ is strictly less than Fbr_{yield} for most engineering materials in use, if we establish a cutoff, at design limit load, of $0.67 \times$ ultimate load so that Fbr applied never exceeds Fbr_{yield}, have we derived an optimally safe "maximum" loading approach on fastener arrays for most engineering materials in common use today?

Conclusions

Given the fact that subassemblies of aircraft, e.g., can have varying degrees of loads placed on them by ever-shifting loads being continuously exerted upon them by changes in "flight conditions" (changes in attitude), what actually takes place in a bolt group is that the most heavily loaded fastener "sheds" that portion of its load that it cannot sustain to the next heaviest loaded fastener, etc.

This shedding process will continue throughout the bolt group until one of three events happens. The bolt group will achieve equilibrium; the bolt group will be knocked out of equilibrium because of a sudden change in loading, thus requiring that a new state of equilibrium be established; or the bolt group will rupture since the load being exerted is beyond the capacity of the fasteners to bear. In this paper, we refer to the notion of "load shedding" as "load rippling," which we feel is an apt description of the internal processes which any bolt group undergoes in continuous load-bearing adjustment to maintain a state of equilibrium.

Acknowledgments

The author would like to extend his sincere appreciation to E. Thomas Matthias for not only providing the unique line equations of Fig. 12, but the concept of ripple effect. Also, a word of appreciation to Alex Lucas who, with Tom, not only provided the essence of the approach, but developed the simplifying assumptions necessary to consider. Both of these gentlemen, past employees of McDonnell Douglas Canada, made it possible to check and compare real and simulated problems with this approach to the conventional approach discussed.

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[2]MIL-HDBK 5E, Figures 3.7.4.1.6 (n), (o), and (p), *FAA Requirements*.